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A Probe Particle in Kerr-Newman-deSitter Cosmos

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We consider the force acting on a spinning charged test particle (probe particle) with the mass m and the charge q in slow rotating the Kerr-Newman-deSitter(KNdS) black hole with the mass M and the charge Q. We consider the case which the spin vector of the probe particle is parallel to the angular momentum vector of the KNdS space-time. We take account of the gravitational spin-spin interaction under the slow rotating limit of the KNdS space-time. When $Q = M$ and $q = m$, we show that the force balance holds including the spin-spin interaction and the motion is approximately same as that of a particle in the deSitter space-time. This force cancellation suggests the possibility of the existence of an exact solution of spinning multi-KNdS black hole.

I. INTRODUCTION

A half of decade ago the multi-black hole solution with a positive cosmological constant has been discovered by Kastor and Traschen [1]. The Kastor-Traschen(KT) solution describes the dynamical system of multi-black holes. The solution is important because it offers a test of the cosmic censorship conjecture [2], a picture of two or several black holes collision [3] and an example supporting the cosmic no-hair conjecture [4]. The issue on the differentiability of the cosmological horizon is also discussed [5].

In this paper we discuss the possibility of the extension into the spinning version of the KT solution. In the limit of the zero cosmological constant, the KT solution becomes the Majumdar-Papapetrou(MP) solution [6]. The existence is closely related to the static force cancellation as $Q = M$. The force balance can be confirmed by a charged probe particle. The spinning version of the MP solution has been constructed by Israel-Wilson [7] and Perjes [8]. Unfortunately, the solution has naked singularities [9] when $Q = M$. On the other hand, in the Kerr-Newman-deSitter(KNdS) space-time, there are cases with no singularities even if $Q = M$. This implies that the exact solution is a spinning multi-*black-hole* solution if there is. This is a great advantage in the physical aspect. It is also important to consider an exact solution in the heterotic string theory on a torus here [10]. In higher dimensions than five, the multi-soliton solution has no singularities and the extreme limit is surprisingly identical with the saturation of the Bogomol'nyi bound [11].

In the spinning case, the force cancellation between a spinning particle and the Kerr-Newman soliton with $Q = M$ has been confirmed up to and including the gravitational spin-spin interaction [12] by using a spinning charged probe particle. The equation of motion was derived in [13] [14] [15]. Hence it is natural to think that the force cancellation including the spin-spin interaction is closely related to the existence of the spinning multi-soliton solution, the IWP exact solution. Moreover, there is another explicit example of force cancellation in the Einstein-Maxwell-dilaton-axion(EMDA) system which describes a low energy string theory [16]. The exact spinning multi-soliton solution definitely exists in the EMDA system [17]. On the other hand, the force cancellation is not guaranteed in the Einstein-Maxwell-dilaton(EMD) system [18] for a probe particle having the same gyromagnetic ratio as that of the background space-time. Although we cannot definitely say, this situation might reflect the fact that the exact spinning multi-soliton solution has not been discovered yet in the EMD system.

In this paper we will show the force cancellation between a spinning test particle and the KNdS space-time in order to search the possibility of the exact spinning multi-black hole solution in asymptotically deSitter space-times.

The rest of this paper is organised as follows. In Sec. II, we review the equation of motion for a spinning charged particle following Dixon [14]. We also derive some features of the probe particle which will be used later. In Sec. III, we calculate the force acting on the probe particle in the slow rotating KNdS space-time. For simplicity we consider the probe particle whose spin vector is parallel to the angular momentum vector of the KNdS space-time. We will confirm that the force cancellation holds including the gravitational spin-spin interaction as $Q = M$ and $q = m$ and show that the probe particle freely moves regardless of the central black hole. In Sec. IV, we will summarise our present study and discuss remaining problems. Some exact expressions of the basic equation are written down in the appendix A. In the appendix B, we give the energy bound argument which the saturation yields $Q = M$.

II. EQUATION OF MOTION

The equation of motion for a spinning charged particle was derived by Dixon [14] based on the Papapetrou's work [13]. The basic equations are*

$$v^\nu \nabla_\nu p^\mu = -\frac{1}{2} R^\mu{}_{\nu\alpha\beta} v^\nu S^{\alpha\beta} + qF^\mu{}_\nu v^\nu + \frac{gq}{4m}\Big(\frac{p \cdot v}{-m}\Big) \nabla^\mu F_{\alpha\beta} S^{\alpha\beta} \tag{2.1}$$

and

$$v^\mu \nabla_\mu S^{\alpha\beta} = p^\alpha v^\beta - p^\beta v^\alpha + \frac{gq}{2m}\Big(-\frac{p \cdot v}{m}\Big)\Big(S^{\mu\alpha} F_\mu{}^\beta - S^{\mu\beta} F_\mu{}^\alpha\Big), \tag{2.2}$$

where $\mu = 0, 1, 2, 3$ and $v^\mu = dx^\mu(\tau)/d\tau$. p^μ and $S^{\mu\nu}$ are the 4-momentum and the angular momentum tensor, respectively. g is the gyromagnetic ratio of the probe particle. " $\cdot$ " denotes the inner product with respect to the metric, that is, $X \cdot Y = g_{\mu\nu} X^\mu Y^\nu$. We adopt the normalisation $v \cdot v = -1$. As we want to set the orbit $x^\mu(\tau)$ to be the center of the mass, we impose the "supplementary condition" such that [15]

$$p^\mu S_{\mu\nu} = 0. \tag{2.3}$$

It is well known in general that p^μ is not parallel to v^μ due to the spin. From Eqs. (2.1) and (2.2) with the condition (2.3), we obtain the relation between v^μ and p^μ,

$$v^\mu \simeq \frac{1}{mf}\Big[p^\mu - \frac{q}{m^2 f^2} S^{\mu\nu} F_{\nu\alpha} p^\alpha \Big(1 - \frac{g}{2} f\Big)\Big] \tag{2.4}$$

in the lowest order needed for the later evaluation of the gravitational spin-spin interaction and the slow rotating limit of the background space-time. In Eq. (2.4), the function, f, is defined by

$$f = \frac{1}{m}\frac{p \cdot p}{v \cdot p}. \tag{2.5}$$

The exact expression of Eq. (2.4) is given in appendix A. Due to the spin and electromagnetic field term, the norm of the momentum is not conserved along the orbit of the probe particle [20],

$$v^\nu \nabla_\nu (p \cdot p) = \frac{gq}{2m}\Big(\frac{p \cdot v}{-m}\Big) p^\mu \nabla_\mu F_{\alpha\beta} S^{\alpha\beta}. \tag{2.6}$$

*One can see the similar derivation in the refs [19].

On the other hand $S^{\mu\nu}S_{\mu\nu}$ is exactly conserved,

$$v^\mu \nabla_\mu (S^{\alpha\beta}S_{\alpha\beta}) = 0. \tag{2.7}$$

The quantity Q_κ defined by

$$Q_\kappa = (p^\mu + qA^\mu)\kappa_\mu + \frac{1}{2}S^{\mu\nu}\nabla_\mu \kappa_\nu, \tag{2.8}$$

is a conserved if κ is a Killing vector and satisfies $\pounds_\kappa A^\mu = 0$ and $\pounds_\kappa F_{\mu\nu} = 0$ since

$$v^\mu \nabla_\mu Q_\kappa = qv_\mu \pounds_\kappa A^\mu + \frac{gq}{4m}\Big(\frac{p\cdot v}{-m}\Big)S^{\mu\nu}\pounds_\kappa F_{\mu\nu}. \tag{2.9}$$

III. A PROBE PARTICLE ON KERR-NEWMAN-DESITTER BLACK HOLE AND FORCE BALANCE

In this section, we confirm the force cancellation including the gravitational spin-spin interaction between a probe particle and the central Kerr-Newman-deSitter(KNdS) black hole. For this purpose we omit the higher order term of the angular momentum. Namely, we take the slow rotating limit and the metric becomes

$$ds^2 = -V(r)dt^2 + \frac{1}{V(r)}dr^2 + r^2 d\Omega_2^2 + 2a(V-1)\sin^2\theta dt d\varphi + O(a^2), \tag{3.1}$$

where a is the parameter of the angular momentum and $V(r) = 1 - 2M/r + Q^2/r^2 - (\Lambda/3)r^2$. $\Lambda = 3H^2$ is a positive cosmological constant. The exact expression is given in the appendix A. The vector potential is

$$A = -\frac{Q}{r}(dt - a\sin^2\theta d\varphi) + O(a^2). \tag{3.2}$$

This system has two Killing vectors $\xi = \partial_t$ and $\psi = \partial_\varphi$. These Killing vectors satisfy $\pounds_\kappa A^\mu = 0$ and $\pounds_\kappa F_{\mu\nu} = 0$ and thus we have two constants of motion.

The constants of the motion related to the Killing vectors, ξ and ψ, become

$$\begin{aligned} -Q_\xi &= -\xi^\mu(p_\mu + qA_\mu) - \frac{1}{2}S^{\mu\nu}\nabla_\mu \xi_\nu \\ &= Vp^0 - a(V-1)\sin^2\theta p^\varphi + \frac{qQ}{r} - \frac{aS}{r^2}(V-1)\cos^2\theta \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} Q_\psi &= \psi^\mu(p_\mu + qA_\mu) + \frac{1}{2}S^{\mu\nu}\nabla_\mu \psi_\nu \\ &= a(V-1)\sin^2\theta p^0 + r^2\sin^2\theta p^\varphi + \frac{aqQ}{r}\sin^2\theta + S\cos^2\theta, \end{aligned} \tag{3.4}$$

respectively. Now we assume $v^\theta = p^\theta = 0$. Setting $Q_\xi = -m$ and $Q_\psi = S\cos^2\theta$, p^0 and p^φ can be written as

$$p^0 = \frac{m}{V}\Big(1 - \frac{qQ}{mr}\Big) \quad \text{and} \quad p^\varphi = -\frac{aqQ}{r^3} - \frac{ma(V-1)}{r^2V}\Big(1 - \frac{qQ}{mr}\Big). \tag{3.5}$$

For simplicity we also assume that the spin vector $\vec{S}$ of the probe particle is parallel to that of the background space-time, $\vec{S} = (0,0,S)$ in a sort of Cartesian coordinate. In the present coordinate of Eq. (3.1), each components of the spin tensor $S^{\mu\nu}$ are written as

$$S^{\theta\varphi} = \frac{\cos\theta}{\sin\theta}\frac{S}{r^2}$$

and $S^{r\theta} = S^{r\varphi} = 0$. In this case, Eq. (2.6) becomes

$$v^\mu \nabla_\mu (p\cdot p) = \frac{gq}{m}\Big(\frac{p\cdot v}{-m}\Big)p^\mu \nabla_\mu F_{\theta\varphi}S^{\theta\varphi} + O(aS^2) \tag{3.6}$$

and Eq. (2.7) concludes that S is a constant.

Let us determine the reading order of the function, f, defined by Eq. (2.5). First of all, Eq. (2.4) yields

$$v^0 = \frac{1}{mf}p^0, \quad v^r = \frac{1}{mf}p^r, \quad v^\theta = \frac{1}{mf}p^\theta + O(a^2), \quad v^\varphi = \frac{1}{mf}p^\varphi + O(a^2 S) \tag{3.7}$$

which implies

$$p^\mu p_\mu = (mf)^2 v_\mu v^\mu + O(a^2) = -(mf)^2 + O(a^2). \tag{3.8}$$

Substituting these approximate expression into Eq. (3.6), we obtain the equation for the function f:

$$v^\mu \partial_\mu \ln f^2 = -\frac{gaSqQ}{m^2} v^\mu \partial_\mu \Big(\frac{1}{r^3}\Big) \cos^2\theta + O(a^2), \tag{3.9}$$

and then

$$\begin{aligned} f &\simeq e^{-\frac{gaSqQ}{m^2 r^3}\cos^2\theta} \\ &\simeq 1 - \frac{gaSqQ}{m^2 r^3} \sin\theta\cos\theta =: 1 - g\tilde{f}, \end{aligned} \tag{3.10}$$

where $\tilde{f} = \frac{aSqQ}{m^2 r^3}\cos^2\theta$.

Now we can calculate the force acting on the probe particle. Noting

$$\begin{aligned} v^\mu \partial_\mu p^r &= m v^\mu \partial_\mu (f v^r) \\ &= mf v^\mu \partial_\mu v^r + m(v^r)^2 \partial_r f, \end{aligned} \tag{3.11}$$

we see that the equation of motion becomes

$$mf v^\mu \partial_\mu v^r = -m(v^r)^2 \partial_r f - \Gamma^r_{\alpha\beta} v^\alpha p^\beta - \frac{1}{2} R^r{}_{\nu\alpha\beta} v^\nu S^{\alpha\beta} + qF^r{}_\nu v^\nu + \frac{gq}{4m}\Big(-\frac{p\cdot v}{m}\Big)\nabla^r F_{\alpha\beta} S^{\alpha\beta} =: mfF^r, \tag{3.12}$$

where F^μ denotes the force per unit mass. Using Eqs. (3.7) $\sim$ (3.10), each terms in the right-hand side are evaluated as follows,

$$-m(v^r)^2 \partial_r f \simeq gm(1-V)\partial_r \tilde{f} \simeq -\frac{gaSqQ}{r^2 m}\Lambda\cos^2\theta \tag{3.13}$$

$$-f^{-1}\Gamma^r_{\mu\nu} v^\mu p^\nu \simeq -\frac{m}{2}V' = -\frac{Mm}{r^2} + \frac{mQ^2}{r^3} + m\frac{1}{3}\Lambda r \tag{3.14}$$

$$f^{-1} qF^r{}_\mu v^\mu \simeq f^{-1} qVA_0' v^0 \simeq \frac{qQ}{r^2} - \frac{q^2 Q^2}{mr^3} \tag{3.15}$$

$$-\frac{1}{2} f^{-1} R^r{}_{\mu\alpha\beta} v^\mu S^{\alpha\beta} \simeq -R^r{}_{0\theta\varphi} v^0 S^{\theta\varphi} = \frac{6aSM}{r^4}\cos^2\theta \tag{3.16}$$

$$\frac{gq}{4m} f^{-1}\Big(\frac{p\cdot v}{-m}\Big)\nabla^r F_{\alpha\beta} S^{\alpha\beta} \simeq \frac{gq}{2m}\nabla^r F_{\theta\varphi} S^{\theta\varphi} \simeq -V\frac{3gaSqQ}{mr^4}\cos^2\theta \simeq -\frac{3gaSqQ}{mr^4}\cos^2\theta + \frac{gaSqQ}{r^2 m}\Lambda\cos^2\theta \tag{3.17}$$

Summing up Eqs. (3.13)$\sim$(3.17), we obtain the total force,

$$F^r = \frac{d^2 r}{d\tau^2} \simeq \frac{1}{3}\Lambda r - \frac{Mm - Qq}{mr^2} + \frac{(m^2 - q^2)Q^2}{m^2 r^3} + \frac{3aS(2Mm - gQq)}{m^2 r^4}\cos^2\theta. \tag{3.18}$$

Eq (3.18) obviously yields $F^r \simeq \frac{1}{3}\Lambda r = H^2 r$ as $Q = M, q = m$ and $g = 2$. This expression of the force is same as pure deSitter space-time case. That is, the force cancellation holds including the order of the gravitational spin-spin interaction except for the repulsive force due to the cosmological constant. We can show that F^θ and F^φ are negligible under the approximation used here.

In the asymptotically deSitter space-times, it is, however, not trivial to estimate the gyromagnetic ratio, $g_{\rm BG}$, of a background space-time because the metric component $g_{t\varphi}$, which is related to the angular momentum, contains the cosmological constant and there is not the adequate definition of the total angular momentum J. If J is physically defined, the gyromagnetic ratio will be given by $g_{\rm BG} = 2Ma/J$. Subtracting the cosmological constant term from the definition of the total angular momentum, $(1/16\pi)\int_{S_\infty} dS_{\mu\nu}\nabla^\mu\psi^\nu$ [21], we are resulted in $J = Ma$ and $g_{\rm BG} = 2$.

IV. SUMMARY AND DISCUSSION

In this paper we confirmed the force cancellation including the gravitational spin-spin interaction between a probe particle and the slow rotating Kerr-Newman-deSitter(KNdS) space-time. We remind readers that the KNdS space-time with $Q=M$ is free from naked singularities in the slow rotating limit. The force balance holds when $Q=M$, $q=m$ and $g=2$. As we said, this fact may suggest the existence of the exact solution which is the spinning version of the Kastor-Traschen solution. It is important to remind you that one of the present authors found a new exact solution which is the spinning dilatonic version of the Kastor-Traschen solution in the Einstein-Maxwell-dilaton-axion theory with a positive cosmological constant [27].

Even in asymptotically deSitter space-times, the force balance indicates the existence of a symmetry which might be slightly related to supersymmetry. The symmetry should give the relation between components of the metric and the electromagnetic field and help us to solve the Einstein equation. More precisely, the energy bound as the Bogomol'nyi bound holds and a symmetry appears when the bound is saturated. The saturation yields the Killing spinor like equation related to supersymmetry and the equation gives a relation between components. In fact, we can easily show the bound nature in certain case without magnetic field as it is shown in the appendix B.

The definition of the total angular momentum is also important in asymptotically deSitter space-times and should be investigated although it is not directly related to our main aim.

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APPENDIX A: SOME EXACT EXPRESSIONS

The relation between v^μ and p^μ is derived by $v^\alpha \nabla_\alpha (p^\mu S_{\mu\nu})=0$. After some tedious calculation, we obtain

$$v^\mu = \frac{p\cdot v}{p\cdot p}\Big[p^\mu + \frac{1}{(-p\cdot p)}\frac{\frac{1}{2}R_{\nu\rho\alpha\beta}S^{\alpha\beta}p^\rho S^{\mu\nu} - q\Big(1-\frac{g}{2m}\frac{p\cdot p}{p\cdot v}\Big)F_{\alpha\beta}p^\beta S^{\mu\alpha} - gq\Big(\frac{p\cdot v}{-m}\Big)\nabla_\nu F_{\alpha\beta}S^{\alpha\beta}S^{\mu\nu}}{1+\frac{1}{(-p\cdot p)}\Big(R_{\mu\nu\alpha\beta}S^{\mu\nu}S^{\alpha\beta} - \frac{g}{2}F_{\mu\nu}S^{\mu\nu}\Big)}\Big]. \tag{A1}$$

and yields Eq. (2.4) under an appropriate approximation.

Next, we describes the exact expression of the KNdS space-time [22] [23]. The metric and the vector potential are

$$ds^2 = \frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{1+\frac{\Lambda}{3}a^2\cos^2\theta}d\theta^2 + \Big(1+\frac{\Lambda}{3}a^2\cos^2\theta\Big)\frac{a^2\sin^2\theta}{\chi^4\rho^2}\Big(dt - \frac{\sigma^2}{a}d\phi\Big)^2 - \frac{\Delta}{\chi^4\rho^2}(dt - a\sin^2\theta d\phi)^2 \tag{A2}$$

and

$$A = -\frac{Qr}{\chi^2\rho^2}(dt - a\sin^2\theta d\phi), \tag{A3}$$

where

$$\begin{aligned}
\sigma^2 &= a^2 + r^2\\
\Delta &= (a^2+r^2)\Big(1-\frac{\Lambda}{3}r^2\Big) - 2Mr + Q^2\\
\rho^2 &= r^2 + a^2\cos^2\theta\\
\chi^2 &= 1 + \frac{\Lambda}{3}a^2.
\end{aligned}$$

Up to the order of $O(a)$ we obtain Eqs. (3.1) and (3.2).

APPENDIX B: ENERGY BOUND THEOREM IN A CERTAIN CASE

This appendix is based on the Refs. [24] [25] [26]. For simplicity, we consider the case in which the magnetic component is absent. However, we guess that the extension into the case with the magnetic field as well as the electric field is easy task. We define the derivative operator on a spinor ϵ as

$$\hat{\nabla}_i\epsilon = \Big(D_i + \frac{1}{2}K_{ij}\gamma^j\gamma^{\hat{0}} + \frac{i}{2}H\gamma_i - \frac{i}{2}\gamma^{\hat{0}}\gamma^j\gamma_i E_j\Big)\epsilon, \tag{B1}$$

where $i = 1, 2, 3$, $D_i\epsilon = (\partial_i + {}^{(3)}\Gamma_i)\epsilon$ and ${}^{(3)}\Gamma_i = -(1/8)e^{j\hat{k}}D_i e_j^{\hat{\ell}}[\gamma_{\hat{\ell}}, \gamma_{\hat{k}}]$. E^i is the electric field vector. $e_j^{\hat{i}}$, D_i and K_{ij} are a unit-orthogonal basis, a covariant derivative and the extrinsic curvature of three dimensional spacelike hypersurfaces, respectively. The expression of Eq. (B1) is inspired by $N = 2$ supergravity with a *negative* cosmological constant. Although the derivative operator of Eq. (B1) is not covariant form in the full space-time, it is enough to argue the energy bound in the present situation. ϵ is assumed to satisfy the modified Witten equation

$$\gamma^i\hat{\nabla}_i\epsilon = 0. \tag{B2}$$

In the same way as [24] [25] [26], we obtain the identity

$$\begin{aligned} D^i(\epsilon^\dagger\hat{\nabla}_i\epsilon) = (\hat{\nabla}_i\epsilon)^\dagger(\hat{\nabla}^i\epsilon) &+ \frac{1}{4}\epsilon^\dagger\Big[{}^{(3)}R + K^2 - K_{ij}K^{ij} - 6H^2\Big]\epsilon + \frac{1}{2}\epsilon^\dagger D_i(K^i_j - \delta^i_j K)\gamma^j\gamma^{\hat{0}}\epsilon \\ &- \frac{1}{2}\epsilon^\dagger E^i E_i\epsilon - i\epsilon^\dagger D^i E_i\gamma^{\hat{0}}\epsilon. \end{aligned} \tag{B3}$$

Taking the volume integral of the above and using a parts of the Einstein and Maxwell equations,

$$\int_{S_\infty} dS_i\epsilon^\dagger\hat{\nabla}_i\epsilon = \int_V dV\Big[(\hat{\nabla}_i\epsilon)^\dagger(\hat{\nabla}^i\epsilon) + 4\pi\epsilon^\dagger(T_{\hat{0}\hat{0}} + T_{\hat{0}\hat{i}}\gamma^{\hat{i}}\gamma^{\hat{0}})\epsilon + 4\pi i\epsilon^\dagger\rho_e\gamma^{\hat{0}}\epsilon\Big], \tag{B4}$$

where $T_{\mu\nu}$ is the energy momentum tensor subtracted by the energy of the electric field and ρ_e is the electric charge density. The left-hand side of Eq. (B4) can be written as

$$\int_{S_\infty} dS^i\epsilon^\dagger\hat{\nabla}_i\epsilon = \int_{S_\infty} dS^i\epsilon_0(\Gamma'_i - \gamma_i\gamma^j\Gamma'_j)\epsilon_0, \tag{B5}$$

where

$$\Gamma'_i = {}^{(3)}\Gamma_i + \frac{1}{2}H\gamma_i - \frac{i}{2}\gamma^{\hat{0}}\gamma^j\gamma_i E_j \tag{B6}$$

and ϵ_0 is a constant spinor satisfying $\gamma^{\hat{0}}\epsilon_0 = -i\epsilon_0$. Inserting Eq. (B6) into Eq. (B5), we obtain

$$\int_{S_\infty} dS^i\epsilon^\dagger\hat{\nabla}_i\epsilon = \frac{1}{4}\int_{S_\infty} dS^i\epsilon_0(\partial_j h_i{}^j - \partial_i h^j{}_j)\epsilon_0 + \frac{1}{2}\int_{S_\infty} dS_i\epsilon_0^\dagger(K^i{}_j - \delta^i_j K + 2H\delta^i_j)\gamma^j\gamma^{\hat{0}}\epsilon_0 - \int_{S_\infty} dS_i\epsilon_0^\dagger E^i\epsilon_0, \tag{B7}$$

where $h_{\mu\nu} = g_{\mu\nu} - \overline{g}_{\mu\nu}$ and $\overline{g}_{\mu\nu}$ is the metric of the deSitter space-time. The first and second term in the right-hand side are exactly the ADM energy and the net 3-momentum. The last term is the total electric charge. Note that we have to take the asymptotic region carefully. The situation differs from asymptotic flat case in which the extrinsic curvature of the spacelike hypersurface behaves $K^i_j \to 0$ towards the spatial infinity. In asymptotically deSitter space-times, the slices with $K \to 3H$ as $r \to \infty$ are best to define the conserved charges [26]. Moreover, we note that the existence of the solution for the modified Witten equation is guaranteed and the surface integral $\int_{S_\infty} dS_i\epsilon^\dagger\hat{\nabla}^i\epsilon$ can be written as Eq. (B5) if the traceless part of the extrinsic curvature has the behaviour $\tilde{K}^i_j = O(1/r^2)$ near the infinity. This implies that the second term in the right-hand side of Eq. (B7) vanishes [26].

Finally, Eqs. (B4) and (B7) lead us the result of the inequality $M_{\rm ADM} \geq |Q|$ under the dominant energy condition and the causality condition of the electric current. The saturation yields $\hat{\nabla}_i\epsilon = 0$ slightly related to $N = 2$ supersymmetry. As the Majumdar-Papapetrou solution [28], the Kastor-Traschen solution has the super covariantly constant spinor ϵ_0 satisfying $\gamma^{\hat{0}}\epsilon_0 = -i\epsilon_0$. This also strongly suggests the existence of multi-spinning black-hole solution in asymptotically deSitter space-times if we can probe the general energy bound argument including the magnetic field.

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